A time varying speed of light as a solution to cosmological puzzles

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We consider the cosmological implications of light travelling faster in the early Universe. We propose a prescription for deriving corrections to the cosmological evolution equations while the speed of light c is changing. We then show how the horizon, flatness, and cosmological constant problems may be solved. We also study cosmological perturbations in this scenario and show how one may solve the homogeneity and isotropy problems. As it stands, our scenario appears to most easily produce extreme homogeneity, requiring structure to be produced in the Standard Big Bang epoch. Producing significant perturbations during the earlier epoch would require a rather careful design of the function $c(t)$. The large entropy inside the horizon nowadays can also be accounted for in this scenario.

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I. THE PUZZLES OF THE BIG BANG MODEL

Cosmologists have long been dissatisfied with the "Standard Big Bang" (SBB) model of the Universe. This is not due to any conflict between the big bang theory and observations, but because of the limited scope offered by the SBB to explain certain striking features of the Universe. From the SBB perspective the homogeneity, isotropy, and "flatness" of the Universe, and the primordial seeds of galaxies and other structure are all features which are "built in" from the beginning as initial conditions. Cosmologists would like to explain these features as being the result of calculable physical processes. A great attraction of the Inflationary Cosmologies [1] is that they address these issues by showing on the basis of concrete calculations that a wide variety of initial conditions evolve, during a period of cosmic inflation, to reflect the homogeneity, isotropy, flatness and perturbation spectrum that we observe today.

So far, *all* attempts to achieve this kind of improvement over the SBB have wound up taking the basic inflationary form, where the observable Universe experiences a period of "superluminal" expansion. This is accomplished by modifying the matter content of the Universe in such a way that ordinary Einstein gravity becomes repulsive and drives inflationary expansion. This process is in many ways remarkably straightforward and has found numerous realizations over the years ([2–5], etc), although it might still be argued that a truly compelling microscopic foundation for inflation has yet to emerge.

One interesting question is whether inflation is the *right* solution to the cosmological puzzles. Is inflation really what nature has chosen to do? When this matter is discussed there is a notable absence of any real competition to inflation, and this must be counted in inflation's favour. However, we believe the picture would become much clearer if some kind of debate along these lines were possible. To this end, we discuss here a possible alternative to inflationary cosmology which, while not as well developed as today's inflationary models, might lead to some illuminating discussion.

In this alternative picture, rather than changing the matter content of the Universe, we change the speed of light in the early Universe. We assume that the Universe matter content is the same as in the SBB, that is, the Universe is radiation dominated at early times. We also assume that Einstein's gravity is left unchanged, in a sense made precise in Section IV. The geometry and expansion factor of the Universe are therefore the same as in the SBB. However the *local* speed of light, as measured by free falling observers associated with the cosmic expansion, varies in time, decelerating from a very large value to its current value.

We discuss below how Varying Speed of Light (VSL) models might resolve the same cosmological puzzles as inflation, and offer a resolution to the cosmological constant problem as well. We shall not dwell on the possible mechanisms by means of which the speed of light could have changed. Rather we wish to concentrate on the conditions one should impose on VSL models for their cosmological implications to be interesting. This phenomenological approach should be regarded as a curiosity, which, we hope, will prompt further work towards an actual theory in which the physical basis of VSL models is realized.

One may doubt that such a self-consistent theory could ever be constructed. We therefore feel forced to transcend the scope of this paper, and discuss essential aspects of such a theory. We find it befitting to start our discussion with an assessment of the experimental meaning of a varying c (Section II). We also need to be more specific about VSL theories in order to tackle the flatness, cosmological constant, homogeneity, and entropy problems. In Section IV we state what is actually required from any VSL theory to solve these problems. However in Appendix I we lay out the foundations for such a theory.

II. THE MEANING OF A VARIABLE SPEED OF LIGHT

We first address the question of the meaning of a varying speed of light. Could such a phenomenon be proved or disproved by experiment? *Physically* it does not make sense to talk about constancy *or* variability of any dimensional "constant". A measurement of a dimensional quantity must always represent its ratio to some standard unit. For example, the length of my arm in meters is really the dimensionless quantity given by the ratio of the arm length to the length of a meter stick. If the ratio varied, one *could* interpret this as a variation in either (or both) of the two lengths. In familiar situations, there is usually a preferred interpretation which distinguishes itself by giving a simpler view of the world. Choosing a given person's arm as a standard of length would require a whole range of simple objects to undergo peculiar dynamics, whereas assuming the meter stick to be constant would usually give a much simpler picture.

None the less, a given theory of the world requires dimensional parameters. If these parameters varied, how would that process show up in experiments? Suppose we set out to measure the speed of light. For this one needs a length measure (rod) and a clock. In a world described by a theory with time varying dimensional parameters, it is quite possible that the rods and clocks, as well as the photon speeds, could all vary. Because measurements are fundamentally dimensionless, the experimental result will only measure some dimensionless combination of the fundamental constants. Let us sketch a simple illustration: Suppose we measure time with an atomic clock. Taking the Rydberg energy ($E_R = m_e e^4/2(4\pi\epsilon_o)^2\hbar^2$) to represent the dependence of all atomic energy levels on the fundamental constants, the oscillation period of the atomic clock will be $\propto \hbar/E_R$. Likewise, taking the Bohr radius ($a_0 = 4\pi\epsilon_0\hbar^2/m_e e^2$) to reflect the relationship between the lengths of ordinary objects (made of atoms) and the fundamental constants, the length of our rod is $\propto a_0$. Thus a measurement of c with our equipment is really a measurement of the dimensionless quantity

$$\frac{c}{a_0/(\hbar/E_R)} = \frac{8\pi\epsilon_0}{\alpha} \tag{1}$$

essentially the fine structure constant. We could of course use other equipment which depends in different ways on the fundamental dimensionless constants. For example, pendulum clocks will necessarily involve Newton's constant G. Different experiments will result, which measure different dimensionless combinations of the fundamental dimensional constants. Our conclusion that physical experiments are only sensitive to dimensionless combinations of dimensional constants is hardly a new one. This idea has been often stressed by Dicke (eg. [21]), and we believe this is not controversial.

Thus, speaking in theoretical terms of time varying dimensional constants can lead to problems. To give an historical example, papers [18,19] were written claiming stringent experimental upper bounds on the time variability of the dimensional quantity $\hbar c$. In these the product $E\lambda$ was found to be the same for light emitted at very different redshifts. From the deBroglie relation $\hbar c = E\lambda$ one infers the constancy of $\hbar c$. Bekenstein gives an illuminating discussion of the fallacy built into this argument [20]. Built into $E \propto 1/a$ and $\lambda \propto a$ is the assumption that $\hbar c$ is constant, for otherwise the wavevector k^μ and the momentum vector p^μ could not both be parallel transported. Hence the experimental statement that $\hbar c$ is constant is circular.

What would we do therefore if we were to observe changing dimensionless quantities? Any theory explaining the phenomenon would necessarily have to make use of dimensional quantities. It would a priori be a matter of choice, prejudice, or convenience to decide which dimensional quantities are variable and which are constant (as we mentioned in the illustration above). There would be a kind of equivalence, or duality between theories based on any two choices as far as dimensionless observations are concerned. However, the equations for two theories which are observationally equivalent, but which have different dimensional parameters varying, will in general not look the same, and again simplicity will end up being an important factor in making a choice between theories. In what follows, we will prefer to work with models which have the simplicity of "minimal coupling".

Let us illustrate this point with a topical example. There has been a recent claim [22] of experimental evidence for a time changing fine structure constant $\alpha = e^2/(4\pi\hbar c)$. Although the ongoing chase for systematics precludes any definitive conclusions, let us assume for the purpose of the argument that the effect is real.

In building a theory which explains a variable α we must make a decision. We could *postulate* that electric charge changes in time, or, say, that $\hbar c$ must change in time. Bekenstein [23] constructs a theory based on the first alternative. He postulates a Lorentz invariant action, which does not conserve electric charge. Our theory is based on the second choice. We postulate breaking Lorentz invariance, a changing $\hbar c$, and consequently non-conservation of energy. Any arguments against the experimental meaning of a changing c can also be directed at Bekensteins' changing e theory, and such arguments are in both cases meaningless. In both cases the choice of a changing dimensional "constant" reverts to the postulates of the theory and is not, a priori, an experimental issue. The observables are always dimensionless. However, the *minimally coupled* theories based on either choice are *not* dual (as we shall point out in Appendix I). For this reason one might prefer one formulation over the other.

Finally, and on a different tone, suppose that future experiments were to confirm that not only α changes in time, but also that there are time variations in dimensionless coupling constants based on other interactions, $\alpha_i = g_i^2/(\hbar c)$[1]. Suppose further that the ratios between the various constants, $r_{ij} = \alpha_i/\alpha_j$, were observed to be

[1]In writing these constants we have assumed that the couplings of these interactions are defined in terms of "charges" (with dimensions of $[E]^{1/2}[L]^{1/2}$).

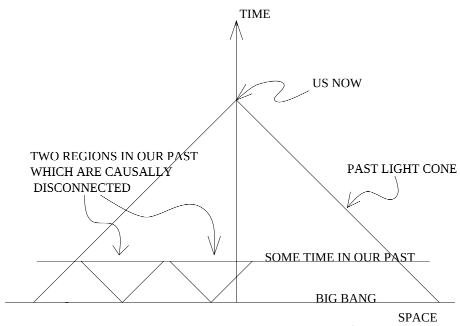

FIG. 1. Conformal diagram (light at 45°) showing the horizon structure in the SBB model. Our past light cone contains regions outside each others' horizon.

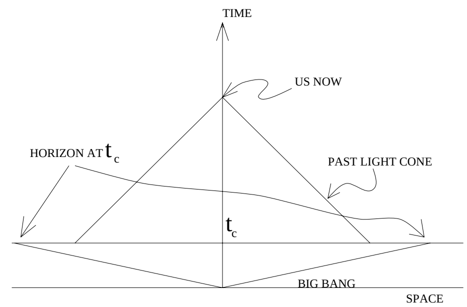

FIG. 2. Diagram showing the horizon structure in a SBB model in which at time t_c the speed of light changed from c^- to $c^+ \ll c^-$. Light travels at 45° after t_c but it travels at a much smaller angle with the space axis before t_c. Hence it is possible for the horizon at t_c to be much larger than the portion of the Universe at t_c intersecting our past light cone. All regions in our past have then always been in causal contact.

constant. Choosing what dimensional constants were indeed constants would still be a matter of taste. One could still define a theory in which the various charges g_i change in time, with fixed ratios, and $\hbar c$ remains constant. However it would perhaps start to make more sense, merely for reasons of simplicity, to postulate instead a changing $\hbar c$.

Therefore, even though a variable c cannot be made a dimensionless statement, evidence in favour of theoretical models with varying c could be accrued if the other α_i changed, with fixed ratios.

III. COSMOLOGICAL HORIZONS

Perhaps the most puzzling feature of the SBB is the presence of cosmological horizons. At any given time any observer can only see a finite region of the Universe, with comoving radius $r_h = c\eta$, where η denotes conformal time, and c the speed of light. Since the horizon size increases with time we can now observe many regions in our past light cone which are causally disconnected, that is, outside each others' horizon (see Fig. 1). The fact that these regions have the same properties (eg. Cosmic Microwave background temperatures equal to a few parts in 10^5) is puzzling as they have not been in physical contact. This is a mystery one may simply relegate to the setting up of initial conditions in our Universe.

One may however try to explain these very peculiar initial conditions. The horizon problem is solved by inflationary scenarios by postulating a period of accelerated or superluminal expansion, that is, if a is the expansion factor of the Universe, a period with $\ddot{a} > 0$. The Friedman equations require that the strong energy condition $\rho + 3p/c^2 \geq 0$ must then be violated, where ρc^2 and p are the energy density and pressure of the cosmic matter. This violation is achieved by the inflaton field. If $\ddot{a} > 0$ for a sufficiently long period one can show that cosmological horizons are a post-inflation illusion, and that the whole observed Universe has in fact been in causal contact since an early time.

A more minimalistic way of solving this problem is to postulate that light travelled faster in the Early Universe. Suppose there was a "phase transition" at time t_c when the speed of light changed from c^- to c^+. Our past light cone intersects $t = t_c$ at a sphere with comoving radius $r = c^+(\eta_0 - \eta_c)$, where η_0 and η_c are the conformal times now and at t_c. This is as much of the Universe after the phase transition as we can see today [6]. On the other hand the horizon size at t_c has comoving radius $r_h = c^-\eta_c$. If $c^-/c^+ \gg \eta_0/\eta_c$, then $r \ll r_h$, meaning that the whole observable Universe today has in fact always been in causal contact (see Fig. 2). Some simple manipulations show that this requires

$$\log_{10} \frac{c^-}{c^+} \gg 32 - \frac{1}{2}\log_{10} z_{eq} + \frac{1}{2}\log_{10} \frac{T_c^+}{T_P^+} \tag{2}$$

where z_{eq} is the redshift at matter radiation equality, and T_c^+ and T_P^+ are the Universe and the Planck temperatures after the phase transition. If $T_c^+ \approx T_P^+$ this implies light travelling more than 30 orders of magnitude faster before the phase transition. It is tempting, for symmetry reasons, simply to postulate that $c^- = \infty$ but this is not strictly necessary.

IV. A PRESCRIPTION FOR MODIFYING PHYSICAL LAWS WHILE THE SPEED OF LIGHT IS VARYING

Hidden in the above argument is the assumption that the geometry of the Universe is not affected by a changing c. We have allowed a changing c to do the job normally done by "superluminal expansion". To enhance this effect we have forced the geometry to still be the SBB geometry. We now elaborate on this assumption. We will propose a prescription for how, in general, to modify gravitational laws while c is changing. This prescription is merely the one we found the most fertile. In Appendix I we describe in detail a theory which realizes this prescription.

The basic assumption is that a variable c does not induce corrections to curvature in the cosmological frame, and that Einstein's equations, relating curvature to stress energy, are still valid. The rationale behind this postulate is that c changes in the local Lorentzian frames associated with cosmological expansion. The effect is a special relativistic effect, not a gravitational effect. Therefore curvature should not feel a changing c.

The previous statement is not covariant. However introducing a function $c(t)$ is not even Lorentz invariant. So it is not surprising that a favoured gauge, or coordinate choice, must be made, where the function $c(t)$ is specified, and in which the above postulate holds true. The cosmological frame (with the cosmological time t) provides such a preferred frame.

In a cosmological setting the postulate proposed implies that Friedman equations remain valid even when $\dot{c} \neq 0$:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{Kc^2}{a^2} \tag{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3\frac{p}{c^2}\right) \tag{4}$$

where, we recall, ρc^2 and p are the energy and pressure densities, $K = 0, \pm 1$ and G the curvature and the gravitational constants, and the dot denotes a derivative with respect to proper time. If the Universe is radiation dominated, $p = \rho c^2/3$, and we have as usual $a \propto t^{1/2}$. We have assumed that a frame exists where $c = c(t)$, and identified this frame with the cosmological frame.

The assumption that Einstein's equations remain unaffected by decelerating light carries with it an important consequence. Bianchi identities apply to curvature, as a geometrical identity. These then imply stress energy conservation as an integrability condition for Einstein's equations. If $\dot{c} \neq 0$, however, this integrability condition is not stress energy conservation. Source terms, proportional to $\dot{c}/c$, come about in the conservation equations.

Seen in another way, the conservation equations imply an equation of motion for free falling point particles. This is normally the geodesic equation, but now source terms will appear in the geodesic equation. Clearly a violation of the weak equivalence principle is implied while c is changing [7]. This, of course, does not conflict with experiment, as we take $\dot{c} \neq 0$ only in the Early Universe, possibly for only a very short time (such as a phase transition).

Although this is a general remark we shall be concerned mostly with violations of energy conservation in a cosmological setting. Friedman equations can be combined into a "conservation equation" with source terms in $\dot{c}/c$ and $\dot{G}/G$:

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = -\rho\frac{\dot{G}}{G} + \frac{3Kc^2}{4\pi G a^2}\frac{\dot{c}}{c} \tag{5}$$

In a flat Universe ($K = 0$) a changing c does not violate mass conservation. Energy, on the other hand, is proportional to c^2. If, however, $K \neq 0$ not even mass is conserved.

In Eqn. 5 we have included the effects of $\dot{G}$ under the same postulate merely for completness. In such a formulation VSL does not reduce to Brans Dicke theory when $\dot{c} = 0$, and $\dot{G} \neq 0$. This is because we postulate that Friedmann equations remain unchanged, which implies that the conservation equations acquire terms in $\dot{c}$ and $\dot{G}$. In Brans Dicke theory one postulates exactly the opposite: the conservation equations must still be valid, so that the weak equivalence principle is satisfied. While we could have taken this stance for c as well we feel that violation of energy conservation is the hallmark of changing c. Variable c must break Poincare invariance, for which energy is the Noether current. Barrow [25] has proposed a formulation of VSL which has the correct Brans Dicke limit.

V. THE FLATNESS PUZZLE

We now turn to the flatness puzzle. The flatness puzzle can be illustrated as follows. Let ρ_c be the critical density of the Universe:

$$\rho_c = \frac{3}{8\pi G}\left(\frac{\dot{a}}{a}\right)^2 \tag{6}$$

that is, the mass density corresponding to $K = 0$ for a given value of $\dot{a}/a$. Let us define $\epsilon = \Omega - 1$ with $\Omega = \rho/\rho_c$. Then

$$\dot{\epsilon} = (1+\epsilon)\left(\frac{\dot{\rho}}{\rho} - \frac{\dot{\rho}_c}{\rho_c}\right) \tag{7}$$

If $p = w\rho c^2$ (with $\dot{w} = 0$), using Eqns.(3), (4), and (5) we have:

$$\frac{\dot{\rho}}{\rho} = -3\frac{\dot{a}}{a}(1+w) - \frac{\dot{G}}{G} + 2\frac{\dot{c}}{c}\frac{\epsilon}{1+\epsilon} \tag{8}$$

$$\frac{\dot{\rho}_c}{\rho_c} = -\frac{\dot{a}}{a}(2 + (1+\epsilon)(1+3w)) - \frac{\dot{G}}{G} \tag{9}$$

and so

$$\dot{\epsilon} = (1+\epsilon)\epsilon\frac{\dot{a}}{a}(1+3w) + 2\frac{\dot{c}}{c}\epsilon \tag{10}$$

In the SBB ϵ grows like a^2 in the radiation era, and like a in the matter era, leading to a total growth by 32 orders of magnitude since the Planck epoch. The observational fact that ϵ can at most be of order 1 nowadays requires that either $\epsilon = 0$ strictly, or an amazing fine tuning must have existed in the initial conditions ($\epsilon < 10^{-32}$ at $t = t_P$). This is the flatness puzzle.

The $\epsilon = 0$ solution is in fact unstable for any matter field satisfying the strong energy condition $1 + 3w > 0$. Inflation solves the flatness problem with an inflaton field which satisfies $1 + 3w < 0$. For such a field ϵ is driven towards zero instead of away from it. Thus inflation can solve the flatness puzzle.

As Eqn. 10 shows a decreasing speed of light ($\dot{c}/c < 0$) would also drive ϵ to 0. If the speed of light changes in a sharp phase transition, with $|\dot{c}/c| \gg \dot{a}/a$, we can neglect the expansion terms in Eqn. 10. Then $\dot{\epsilon}/\epsilon = 2\dot{c}/c$ so that $\epsilon \propto c^2$. A short calculation shows that the condition (2) also ensures that $\epsilon \ll 1$ nowadays, if $\epsilon \approx 1$ before the transition.

The instability of the $K \neq 0$ Universes while $\dot{c}/c < 0$ can be expected simply from inspection of the non conservation equation Eq. (5). Indeed if ρ is above its critical value, then $K = 1$, and Eq. (5) tells us that mass is taken out of the Universe. If $\rho < \rho_c$, then $K = -1$, and then mass is produced. Either way the mass density is pushed towards its critical value ρ_c. In contrast with the Big Bang model, during a period with $\dot{c}/c < 0$ only the $K = 0$ Universe is stable.

Note that with the set of assumptions we have used a changing G cannot solve the flatness problem (cf. [8–10]).

We have assumed in the previous discussion that we are close, but not fine-tuned, to flatness before the transition. It is curious to note that this need not be the case. Suppose instead that the Universe acquires "natural initial conditions" (eg. $\epsilon \approx 1$) well before the phase transition occurs. If such Universes are closed they recollapse before the transition. If they are open, then they approach $\epsilon = -1$. This is the Milne Universe, which is our case (constant G) may be seen as Minkowski space-time. Such a curvature dominated Universe is essentially empty, and a coordinate transformation can transform it into Minkowski space-time. Inflation cannot save these empty Universes, as can be seen from Eqn. 10. Indeed even if $1 + 3w < 0$ the first term will be negligible if $\epsilon \approx -1$. This is not true for VSL: the second term will still push an $\epsilon = -1$ Universe towards $\epsilon = 0$.

Heuristically this results from the fact that the violations of energy conservation responsible for pushing the Universe towards flatness do not depend on there being any matter in the Universe. This can be seen from inspection of Eqn. (5).

In this type of scenario it does not matter how far before the transition the "initial conditions" are imposed. We end up with a chaotic scenario in which Darwinian selection gets rid of all the closed Universes. The open Universes become empty and cold. In the winter of these Universes a phase transition in c occurs, producing matter, and leaving the Universe very fine tuned, indeed as an Einstein deSitter Universe (EDSU).

VI. THE COSMOLOGICAL CONSTANT PROBLEM

There are two types of cosmological constant problems, and we wish to start our discussion by differentiating them. Let us write the action as:

$$S = \int dx^4 \sqrt{-g}\left(\frac{c^4(R + 2\Lambda_1)}{16\pi G} + \mathcal{L}_M + \mathcal{L}_{\Lambda_2}\right) \quad (11)$$

where $\mathcal{L}_M$ is the matter fields Lagragian. The term in Λ_1 is a geometrical cosmological constant, as first introduced by Einstein. The term in Λ_2 represents the vacuum energy density of the quantum fields [11]. Both tend to dominate the energy density of the Universe, leading to the so-called cosmological constant problem. However they represent two rather different problems. We shall attempt to solve the problem associated with the first, not the second, term. Ususally one hopes that the second term will be cancelled by an additional couter-term in the Lagrangian. In the rest of this paper it is the geometrical cosmological constant that is under scrutiny.

If the cosmological constant $\Lambda \neq 0$ then the argument in the previous section still applies, with $\rho = \rho_m + \rho_\Lambda$, where ρ_m is the mass density in normal matter, and

$$\rho_\Lambda = \frac{\Lambda c^2}{8\pi G} \quad (12)$$

is the mass density in the cosmological constant. One still predicts $\Omega_m + \Omega_\Lambda = 1$, with $\Omega_m = \rho_m/\rho_c$ and $\Omega_\Lambda = \rho_\Lambda/\rho_c$. However now we also have

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}\left(\rho_m + \frac{p_m}{c^2}\right) = -\dot{\rho}_\Lambda - \rho\frac{\dot{G}}{G} + \frac{3Kc^2}{4\pi G a^2}\frac{\dot{c}}{c} \quad (13)$$

If Λ is indeed a constant then from Eq. (12)

$$\frac{\dot{\rho}_\Lambda}{\rho_\Lambda} = 2\frac{\dot{c}}{c} - \frac{\dot{G}}{G} \quad (14)$$

If we define $\epsilon_\Lambda = \rho_\Lambda/\rho_m$ we then find, after some straightforward algebra, that

$$\dot{\epsilon}_\Lambda = \epsilon_\Lambda\left(3\frac{\dot{a}}{a}(1 + w) + 2\frac{\dot{c}}{c}\frac{1 + \epsilon_\Lambda}{1 + \epsilon}\right) \quad (15)$$

Thus, in the SBB model, ϵ_Λ increases like a^4 in the radiation era, like a^3 in the matter era, leading to a total growth by 64 orders of magnitude since the Planck epoch. Again it is puzzling that ϵ_Λ is observationally known to be at most of order 1 nowadays. We have to face another fine tuning problem in the SBB model: the cosmological constant problem.

If $\dot{c} = 0$ the solution $\epsilon_\Lambda = 0$ is in fact unstable for any $w > -1$. Hence violating the strong energy condition $1 + 3w > 0$ would not solve this problem. Even in the limiting case $w = -1$ the solution $\epsilon_\Lambda = 0$ is not an attractor: ϵ_Λ would merely remain constant during inflation, then starting to grow like a^4 after inflation. Therefore inflation cannot "explain" the small value of ϵ_Λ, as it can with ϵ, unless one violates the dominant energy condition $w \geq -1$.

However, as Eqn. (15) shows, a period with $\dot{c}/c \ll 0$ would drive ϵ_Λ to zero. If the speed of light changes suddenly ($|\dot{c}/c| \gg \dot{a}/a$) then we can neglect terms in $\dot{a}/a$, and so

$$\frac{\dot\epsilon_\Lambda}{\epsilon_\Lambda(1+\epsilon_\Lambda)} = 2\frac{\dot c}{c}\frac{1}{1+\epsilon} \tag{16}$$

which when combined with $\dot\epsilon/\epsilon = 2\dot c/c$ leads to

$$\frac{\epsilon_\Lambda}{1+\epsilon_\Lambda} \propto \frac{\epsilon}{1+\epsilon} \tag{17}$$

The exact constraint on the required change in c depends on the initial conditions in ϵ and ϵ_Λ. In any case once both $\epsilon \approx 1$ and $\epsilon_\Lambda \approx 1$ we have $\epsilon_\Lambda \propto c^2$. Then we can solve the cosmological constant problem in a sudden phase transition in which

$$\log_{10}\frac{c^-}{c^+} \gg 64 - \frac{1}{2}\log_{10} z_{eq} + 2\log_{10}\frac{T_c^+}{T_P^+} \tag{18}$$

This condition is considerably more restrictive than (2), and means a change in c by more than 60 orders of magnitude, if $T_c^+ \approx T_P^+$. Note that once again a period with $\dot G/G$ would not solve the cosmological constant problem.

Equations (10) and (15) are the equations one should integrate to find conditions for solving the flatness and cosmological constant problems for arbitrary initial conditions and with arbitrary curves $c(t)$. They generalize the conditions (2) and (18) which are valid only for a starting point with $\epsilon \approx 1$ and $\epsilon_\Lambda \approx 1$ and for a step function $c(t)$.

As in the case of the flatness problem we do not need to impose "natural initial conditions" ($\epsilon_\Lambda \approx 1$) just before the transition. These could have existed any time before the transition, and the argument would still go through, albeit with a rather different overall picture for the history of the Universe.

If $\epsilon_\Lambda \approx 1$ well before the transition, then the Universe soon becomes dominated by the cosmological constant. We have inflation! The curvature and matter will be inflated away. We end up in a de-Sitter Universe. When the transition is about to occur it finds a flat Universe ($\epsilon = 0$), with no matter ($\rho_m = 0$), and with a cosmological constant. If we rewrite Eqn.(15) in terms of $\epsilon_m = \rho_m/\rho_\Lambda$, for $\epsilon = 0$ and $|\dot c/c| \gg \dot a/a$, we have $\dot\epsilon_m = -2(\dot c/c)(1+\epsilon_m)$. Integrating leads to $1+\epsilon_m \propto c^{-2}$. We conclude that we do not need the presence of any matter in the Universe for a VSL transition to convert a cosmological constant dominated Universe into a EDSU Universe full of ordinary matter. This can be seen from Eqns. (13)-(14). A sharp decline in c will always discharge any vacuum energy density into ordinary matter.

We stress the curious point that in this type of scenario the flatness problem is not solved by VSL, but rather by the period of inflation preceding VSL.

VII. THE HOMOGENEITY OF THE UNIVERSE

Solving the horizon problem by no means guarantees solving the homogeneity problem, that is, the uncanny homogeneity of the currently observed Universe across many regions which have apparently been causally disconnected. Although solving the horizon problem is a necessary condition for solving the homogeneity problem, in a generic inflationary model solving the first causes serious problems in solving the latter. Early causal contact between the entire observed Universe allows equilibration processes to homogenize the whole observed Universe. It is crucial to the inflation picture that before inflation the observable universe in well inside the Jeans length, and thus equilibrates toward a homogeneous state. However no such process is perfect, and small density fluctuations tend to be left outside the Hubble radius, once the Universe resumes its standard Big Bang course. These fluctuations then grow like a^2 during the radiation era, like a during the matter era, usually entailing a very inhomogeneous Universe nowadays. This is a common flaw in early inflationary models [12] which requires additional fine-tuning to resolve.

In order to approach this problem we study in Appendix II the effects of a changing c on the theory of scalar cosmological perturbations [13]. The basic result is that the comoving density contrast Δ and gauge-invariant velocity v are subject to the equations:

$$\Delta' - \left(3w\frac{a'}{a} + \frac{c'}{c}\right)\Delta = -(1+w)kv - 2\frac{a'}{a}w\Pi_T \tag{19}$$

$$\begin{aligned} v' + \left(\frac{a'}{a} - 2\frac{c'}{c}\right)v &= \left(\frac{c_s^2 k}{1+w} - \frac{3}{2k}\frac{a'}{a}\left(\frac{a'}{a} + \frac{c'}{c}\right)\right)\Delta \\ &+ \frac{kc^2 w}{1+w}\Gamma - kc\left(\frac{2/3}{1+w} + \frac{3}{k^2c^2}\left(\frac{a'}{a}\right)^2\right)w\Pi_T \end{aligned} \tag{20}$$

where k is the wave vector of the fluctuations, and Γ is the entropy production rate, Π_T the anisotropic stress, and c_s the speed of sound, according to definitions spelled out in Appendix II.

In the case of a sudden phase transition Eqn. (19) shows us that $\Delta \propto c$, regardless of the chosen equations of state for Γ and Π_T. Hence

$$\frac{\Delta^+}{\Delta^-} = \frac{c^+}{c^-} \tag{21}$$

meaning a suppression of any fluctuations before the phase transition by more than a factor of 10^{-60} if condition (18) is satisfied. The suppression of fluctuations induced by a sudden phase transition in c can be intuitively understood in the same fashion as the solution to the flatness problem. Mass conservation violation ensures that only a Universe at critical mass density is stable, if $\dot c/c \ll 0$. But this process occurs locally, so after the phase transition the Universe should be left at critical density *locally*. Hence the suppression of density fluctuations.

We next need to know what are the initial conditions for Δ and v. Suppose that at some very early time t_i one has $\dot c/c = 0$ and the whole observable Universe nowadays is inside the Jeans length: $\eta_0 \ll c_i\eta_i/\sqrt{3}$. The latter condition is enforced as a byproduct of solving the horizon problem. The whole observable Universe nowadays is then initially in a thermal state. What is more each portion of the Universe can be described by the canonical ensemble and so the Universe is homogeneous apart from thermal fluctuations [14]. These are characterized by the mass fluctuation

$$\sigma_M^2 = \frac{\langle \delta M^2 \rangle}{\langle M \rangle^2} = \frac{4k_b T_i}{M c_i^2} \tag{22}$$

Converted into a power spectrum for Δ this is a white noise spectrum with amplitude

$$P_\Delta(k) = \langle |\Delta(k)^2| \rangle \propto \frac{4k_b T_i}{\rho_i c_i^2} \tag{23}$$

What happens to a thermal distribution, its temperature, and its fluctuations, while c is changing? In thermal equilibrium the distribution function of particle energies is the Planck distribution $P(E) = 1/(e^{E/k_b T} - 1)$, where T is the temperature. When one integrates over the whole phase space, one obtains the bulk energy density $\rho c^2 \propto (k_b T)^4/(\hbar c)^3$. Let us now consider the time when the Universe has already flattened out sufficiently for mass to be approximately conserved. To define the situation more completely, we make two additional microphysical assumptions. Firstly, let mass be conserved also for individual quantum particles, so that their energies scale like $E \propto c^2$. Secondly, we assume particles' wavelengths do not change with c. If homogeneity is preserved, indeed the wavelength is an adiabatic invariant, fixed by a set of quantum numbers, eg: $\lambda = L/n$ for a particle in a box of size L.

Under the first of these assumptions a Planckian distribution with temperature T remains Planckian, but $T \propto c^2$. Under the second assumption, we have $\lambda = 2\pi\hbar c/E$, and so $\hbar/c$ should remain constant. Therefore the phase space structure is changed so that, without particle production, one still has $\rho c^2 \propto (k_b T)^4/(\hbar c)^3$, with $T \propto c^2$. A black body therefore remains a black body, with a temperature $T \propto c^2$. If we combine this effect with expansion, with the aid of Eqn. (5) we have

$$\dot{T} + T\left(\frac{\dot{a}}{a} - 2\frac{\dot{c}}{c}\right) = 0 \tag{24}$$

We can then integrate this equation through the epoch when c is changing to find the temperature T_i of the initial state. This fully fixes the initial conditions for scalar fluctuations, by means of (23).

In the case of a sudden phase transition we have $T^+ = T^- c^{2+}/c^{2-}$, and so

$$\sigma_M^{2-} = \frac{4k_b T^-}{M c^{2-}} = \frac{4k_b T^+}{M c^{2+}} \tag{25}$$

or

$$\Delta^-(k)^2 \approx \frac{4k_b T^+}{\rho^+ c^{2+}} \tag{26}$$

but since $\Delta \propto c$ we have

$$\Delta^+(k) \approx \sqrt{\frac{4k_b T^+}{\rho^+ c^{2+}}} \frac{c^+}{c^-} \tag{27}$$

Even if $T^+ = T_P^+ = 10^{19} Gev$ these fluctuations would still be negligible nowadays. Therefore although the Universe ends up in a thermal state after the phase transition, its thermal fluctuations, associated with the canonical ensemble, are strongly suppressed.

For a more general $c(t)$ function the procedure is as follows. Integrate Eqn. (24) backwards up to a time t_i when $\dot{c} = 0$, to find $T(t_i)$. Give $\Delta(t_i)$ a thermal spectrum of fluctuations, according to (23), with $T(t_i)$. With this initial condition integrate Eqns.(19) and (20) (or even better the second order equation (64) given in Appendix II), to find Δ nowadays.

It is conceivable that a careful design of $c(t)$ would leave fluctuations, once $\dot{c} = 0$ again, with the right amplitude and spectrum to explain structure formation. In particular $c(t)$ may be designed so as to convert a white noise spectrum into a scale-invariant spectrum. However we feel that until a mechanism for inducing $c(t)$ is found such efforts are bound to look ludicrously contrived.

We feel that the power of VSL scenarios is precisely in leaving the Universe very homogenous, after c has stopped changing. This would then set the stage for causal mechanisms of structure formation to do their job [15,16].

VIII. THE ISOTROPY OF THE UNIVERSE

There is a sense in which there is an isotropy problem in the SBB model, similar to the homogeneity problem. We follow closely the remark made in [13], pp.26.

In Appendix III we write down the vector Einstein's equations in the vector gauge, and from them we derive the vorticity "conservation" equation when $\dot{c}/c \neq 0$. If v is the vorticity (defined in Appendix) and Π^T the vector stress, we have:

$$v' + (1 - 3w)\frac{a'}{a} v - 2\frac{c'}{c} v = -\frac{kc}{2} \frac{w}{1+w} \Pi^T \tag{28}$$

In the absence of driving stress, v remains constant during the radiation dominated epoch, and decays like $1/a$ in the matter epoch. In [13] it is further argued that the relevant dimensionless quantity is

$$\omega = \frac{(k/a)v}{(a'/ca)} \propto \frac{1}{a^{(1-9w)/2}} \tag{29}$$

Hence for $w > 1/9$ vorticity grows, leading to a further fine tuning problem.

This is most notably a problem if we accept the Planck equipartition proposal, introduced in [17]. At Planck epoch there would then be a significant vorticity. Depending on how one looks at it, this vorticity would then get frozen in or grow, leading to a very anisotropic Universe nowadays.

Whether or not this is a problem is clearly debatable. In any case either inflation or VSL models could solve this prospective problem. For $w < -1/3$ we have that v decays faster than $1/a^2$. Whatever dimensionless quantity one chooses to look at, vorticity is therefore safely inflated away. If $\dot{c}/c \neq 0$ we have that $v \propto c^2$. Again any primordial vorticity is safely suppressed after a phase transition in c satisfying any of the conditions (2) or (18).

IX. THE ENTROPY PROBLEM AND SETTING THE INITIAL CONDITIONS

Let us first consider the SBB model. Let S_h be the entropy inside the horizon, and $\sigma_h = S_h/k_B$ be its dimensionless counterpart. σ_h is of order 10^{96} nowadays. If we assume that the only scales in the cosmological model are the ones provided by the fundamental constants, then at t_P the temperature is T_P. At Planck time, σ_h (being dimensionless) is naturally of order 1. In the SBB model the horizon distance is $d_h = 2t$ in the radiation dominated epoch, and ignoring mass thresholds $t \propto 1/T^2$. If evolution is adiabatic one then has (in a flat Universe)

$$\sigma_h(t) \approx \sigma_h(t_P)\left(\frac{T_P}{T}\right)^3. \tag{30}$$

Since $\sigma_h(t_P) \sim 1$, one has $\sigma_h(t_0) \sim 10^{96}$. Thus the large entropy inside the horizon nowadays is a reflection of the lack of scales beyond the ones provided by the fundamental constants, the fact that the horizon size is much larger nowadays than at Planck time, and the flatness of the Universe. One may rephrase the horizon and flatness problems in terms of entropy [2]. However if one is willing to accept the horizon structure and flatness of the Universe simply as features of the initial conditions (rather than problems), there is no additional entropy problem.

There *is* a problem that arises if one tries to solve the horizon problem, keeping the adiabatic assumption, by means of superluminal expansion. This blows what at Planck time is a region much smaller than the Planck size into a comoving region containing the whole observable Universe nowadays. This solves the horizon problem. However if evolution is adiabatic such a process implies that $\sigma_h(t_0) \ll 1$. Stated in another way, since the number of particles inside the horizon n_h is of the same order as σ_h, this implies an empty Universe nowadays.

More mathematically, if d_h is the horizon proper distance, one has

$$\frac{\dot{\sigma}_h}{\sigma_h} = \frac{3}{d_h} \tag{31}$$

where we have used $d_h = a\int^t dt'/a$. With any standard matter ($p > -\rho c^2/3$) the horizon grows like t. Accordingly σ_h grows like a power of t. On the other hand the horizon grows faster than t if $p < -\rho c^2/3$: it grows exponentially if $p = -\rho c^2$, and like t^n (with $n > 1$) for $-\rho c^2 < p < -\rho c^2/3$. This provides the inflationary solution to the horizon problem. However in the latter case Eqn. (31) implies that σ_h decreases exponentially, leading to $\sigma_h(t_0) \ll 1$. The way inflation bypasses this problem is by dropping the adiabatic assumption. Indeed during inflation the Universe supercools, and a period of reheating follows the end of inflation[2].

In a VSL scenarios the detailed solution to the entropy problem depends on when and what type of "natural conditions" are given to the pre transition Universe. We first derive equations for the entropy under varying c. From $s = (4/3)\rho c^2/T$, $\rho \propto T^4/(\hbar c)^3$, and from Eqns 5 and 13 we obtain that the entropy of radiation satisfies

$$\frac{\dot{s}}{s} = \frac{3}{4}\frac{\dot{\rho}}{\rho} = -3\frac{\dot{a}}{a} + \frac{3}{2}\frac{\dot{c}}{c}\frac{\epsilon(1+\epsilon_\Lambda)}{1+\epsilon} - \frac{3}{2}\frac{\dot{c}}{c}\epsilon_\Lambda \tag{32}$$

If the Universe is EDSU, there are no violations of mass conservation, and entropy is conserved. However if the Universe is open or has a positive cosmological constant, then we have seen that there is creation of mass. Accordingly there must be creation of particles, and entropy is produced. If the Universe is closed, particles are taken away, and the entropy decreases.

The most suspicious case is therefore if the Universe was Einstein de-Sitter before the phase transition. Let us assume therefore that at $t = t_P^-$ (the Planck time with the constants before the transition) the entropy inside the horizon (which has proper size $c^- t_P^-$) was of order 1. Then the entropy inside the Hubble volume at $t = t_P^+$, before and after the transition, is

$$\sigma_h(t_P^+) = \sigma_h(t_P^-)\left(\frac{c^+ t_P^+}{c^- t_P^-}\right)^3 \left(\frac{a(t_P^-)}{a(t_P^+)}\right)^3 \approx 1 \tag{33}$$

where we have used $t_P^+/t_P^- = (c^-/c^+)^2$. One takes a fraction $(c^+/c^-)^3$ of the horizon volume before the transition to make the Hubble volume after the transition. However the entropy inside the horizon has increased since t_P^+ by the same factor. Therefore entropy conservation in this case does not conflict with $\sigma_h(t_P^+) \approx 1$ after the transition. One way of understanding this is that by imposing flatness from the outset (before the transition) one has already "solved" the entropy problem. Notice that the above argument works for any value of t_c/t_P^+.

Now consider the case where "natural" initial conditions were also imposed at t_P^-, with $\Lambda = 0$. One should have $\epsilon(t_P^-)$ of order 1. We have already discussed how the flatness problem is solved in this case, when large empty curvature dominated universes are filled with a

[2] This issue has been carefully analyzed in the context of inflationary models and models with time varying G in [10]

(nearly perfectly) critical energy density during the transition. Open Universes become very empty, but they are still pushed to EDSU at the transition. One may integrate (32) to find that $s^+/s^- = (1+\epsilon^-)^{-3/4}$. One may also use Eqn. 10 to find that ϵ has evolved since t_P^- to $\epsilon^-(t_P^+) + 1 \approx (a(t_P^-)/a(t_P^+))^2 \approx (t_P^-/t_P^+)^2$, where we have used $a \propto t$ for the Milne Universe. Hence we have that during the transition entropy is produced like $s^+/s^- = (t_P^+/t_P^-)^{3/2} = (c^-/c^+)^3$. Given that $a \propto t$ for such Universes, the entropy before the transition in the proper volume of size $c^+t_P^+$ is

$$S^-(c^+t_P^+) = \left(\frac{c^+t_P^+}{c^-t_P^-}\right)^3 \left(\frac{a(t_P^-)}{a(t_P^+)}\right)^3 \approx \left(\frac{c^+}{c^-}\right)^3 \tag{34}$$

that is there is practically no entropy in relevant volume before the transition. However we have that after the transition

$$\sigma_h(t_P^+) = S^+(c^+t_P^+) = S^-(c^+t_P^+)\left(\frac{c^-}{c^+}\right)^3 \approx 1 \tag{35}$$

In such scenarios the Universe is rather cold and empty before the transition. However the transition itself reheats the Universe. Notice that, like in the first case discussed, the above argument works for any value of t_c/t_P^+.

If at $t = t_P^-$ one also has $\epsilon_\Lambda \approx 1$ then we have a scenario in which the cosmological constant dominates, solves the flatness problem, and is discharged into normal matter. However if $\rho_\Lambda \approx \rho_P^-$ at $t \approx t_P^-$, then whatever the transition time, after the transition the Universe will have a density in normal matter equal to $\rho_m = \rho_P^-$. Hence the Hubble time after the transition will be t_P^-, whatever the actual age of the Universe. One may integrate (32) to find that in this case (setting $\epsilon = 0$) the entropy production during the transition is $s^+/s^- = (1+\epsilon_\Lambda^-)^{3/4}$. In the period between $t = t_P^-$ and the transition, ϵ_Λ increases like a^4, and the entropy density is diluted like $1/a^3$. Hence after the transition the entropy density is what it was at $t = t_P^-$, that is $s^+ \approx 1/L_P^{-3}$. If we now follow the Universe until its Hubble time is t_P^+ (when its density is ρ_P^+) we must wait until the expansion factor has increased by a factor of $(\rho_P^+/\rho_P^-)^{1/4}$. Given that $s \propto 1/a^3$ the entropy density is diluted by a factor of $(\rho_P^+/\rho_P^-)^{3/4}$. Therefore the entropy density when the Hubble time is $t = t_P^+$ is $s \approx 1/L_P^{+3}$. Again the dimensionless entropy inside the Hubble volume, when this has size L_P^+, is of order 1.

Finally it is worth noting that treating the pre-transition universe simply as a Roberston-Walker model is no doubt overly simplistic, and we use it simply as a device to introducing our ideas. We expect that further development of these ideas could result in a radically different view of the pre-transition phase (much as has happened with the inflationary scenario). One interesting observation is that one could avoid having multiple Planck times by considering that $G \propto c^4$. Such assumption would not conflict with the dynamics of flatness and Λ, as shown before, but now $t_P^- = t_P^+$.

X. CONCLUSIONS

We have shown how a time varying speed of light could provide a resolution to the well known cosmological puzzles. These "VSL" models could provide an alternative to the standard Inflationary picture, and furthermore resolve the classical cosmological constant puzzle. At a technical level, the proposed VSL picture is not nearly as well developed as the inflationary one, and one purpose of this article is to stimulate further work on the unresolved technical issues. We are not trying to take an "anti-inflation" stand, but we do strongly feel that broadening the range of possible models of the very early Universe would be very healthy for the field of cosmology, and would ultimately allow us to state in more concrete terms the extent to which one model is preferred.

On a more fundamental level we hope to expand the phenomenological approach presented in this paper into a theory where the concept of (Poincare) symmetry breaking provides the physical basis for VSL. Symmetry breaking is also the central ingredient in causal theories of structure formation. We therefore hope to arrive at a scenario where symmetry breaking provides a complete and consistent complement to the SBB model which can resolve the standard puzzles as well as explain the origin of cosmic structure.

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APPENDIX I: A SPECIFIC REALIZATION OF VSL

In this Appendix we set up a specific VSL theory. We first discuss the simple case of the electrodynamics of the point particle in Minkowski space time. We start from Bekenstein's theory of variable α, and show how a VSL alternative could be set up. We highlight the subtleties encountered in the VSL formulation. We then perform the same exercise with the Einstein-Hilbert action. We briefly consider the dynamics of the field $\psi = c^4$. Finally we cast the key elements of our construction into a body of axioms.

A. Electrodynamics in flat space time

A changing α theory was proposed by Bekenstein [23] based on the postulate of Lorentz invariance. The electrodynamics of a point particle was first analyzed. If Lorentz invariance is to be preserved then the particle mass m and its charge e must be variable. In order to preserve "minimal coupling" (reduction to standard electromagnetism when $\alpha = \text{const}$) one chooses the world line action

$$L = -mc\sqrt{-u^\mu u_\mu} + \frac{e}{c} u^\mu A_\mu \tag{36}$$

with $u^\mu = \dot{x}^\mu$, $g_{\mu\nu} = \eta_{\mu\nu}$, $e = e(x^\mu)$, and $m = m(x^\mu)$. Minimal coupling means simply to take the standard action and replace e and m by variables without breaking Lorentz invariance. e and m must then be scalar functions. This action leads to equation:

$$(m\dot{x}_\mu)^{\cdot} = -m_{,\mu} c^2 + \frac{e}{c} u^\nu F_{\mu\nu} \tag{37}$$

with the electromagnetic field tensor defined as

$$F_{\mu\nu} = \frac{1}{e}(\partial_\mu (eA_\nu) - \partial_\nu (eA_\mu)) \tag{38}$$

The electromagnetic action can therefore be defined as:

$$S_{EM} = \frac{-1}{16\pi} \int d^4x F_{\mu\nu} F^{\mu\nu} \tag{39}$$

Also, the particle action (36) may be written as a Lagrangian density:

$$S_M = \int d^4x \frac{\delta^{(3)}(\mathbf{x} - \mathbf{x}(\tau))}{\gamma} (-mc^2 + (e/c) u^\mu A_\mu) \tag{40}$$

in which γ is the Lorentz factor. Maxwell's equations are then:

$$e\partial_\mu (F^{\mu\nu}/e) = 4\pi j^\mu \tag{41}$$

with the current

$$j^\mu = \frac{\delta^{(3)}(\mathbf{x} - \mathbf{x}(\tau))}{\gamma} \frac{eu^\mu}{c} \tag{42}$$

This current[3] is the current which couples to the gauge field, and in the rest frame it equals e. Therefore it cannot be conserved, and indeed we have that

$$\partial_\mu j^\mu = \frac{j^\mu}{e^2} \partial_\mu e \tag{43}$$

[3] There is an alternative view in which rather than a changing e one considers that the vacuum is a dielectric medium with variable ϵ. One may then identify a conserved charge, but this is not the charge which couples to the gauge field.

Let us now postulate instead that a changing α is to be interpreted as $c \propto \hbar \propto \alpha^{-1/2}$, and that e and m are to be seen as constants. Minimal coupling, in the above sense, would then prompt us to consider the action (36), but with $c = c(x^\mu)$ everywhere, and e and m constants. This action leads to equations:

$$m\ddot{x}_\mu = \frac{1}{2}(mc^2)_{,\mu} + \frac{e}{c} u^\nu F_{\mu\nu} \tag{44}$$

with the electromagnetic tensor defined as

$$F_{\mu\nu} = c(\partial_\mu (A_\nu/c) - \partial_\nu (A_\mu/c)) \tag{45}$$

However the above construction is not complete. In spite of the appearance of Eqns. (36), (44), and (45), Lorentz invariance is broken. This boils down to the fact that, say ∂_μ is no long a 4-vector. Even if c were to be regarded as a scalar, ∂_μ would contain c in its zero component, but not in its spatial components. The usual contractions leading to S could still be taken but S would no longer be a scalar. This manifests itself in the equations (44) in the fact that in $\ddot{x}^\mu$ there are terms in ∂c which break Lorentz invariance.

Since the action is not Lorentz invariant, a minimal coupling prescription cannot possibly be true in every coordinate system. Minimal coupling is now the statement that there is a preferred reference frame in which the action is to be obtained from the standard action simply by replacing c with a field. Let us call this frame the "light frame". In regions in which c changes very little changes in the action upon Lorentz transformations are negligible. Hence all boosts performed upon the light frame become nearly equivalent and Lorentz invariance is recovered.

The Maxwell equations in a VSL theory become

$$\frac{1}{c}\partial_\mu (cF^{\mu\nu}) = 4\pi j^\mu \tag{46}$$

in the light frame. Given that Lorentz invariance is broken, one can no longer expect the general expression for a conserved current to take the form $\partial_\mu j^\mu = 0$. Indeed one could try and compute ∂_ν of equations (46), but now ∂_μ and ∂_ν do not commute. Also their commutator is not Lorentz invariant: for instance $[\partial_0, \partial_i] = (-\partial_i c/c^2)\partial_0$. Still, $\partial_\mu j^\mu = 0$ holds in the time frame. It is just that this expression transforms into something more complicated in other frames. The more complicated expression would still place constraints on the theory, which could still be called "conservation of charge".

B. Minimal coupling to gravity

Let us now examine gravity in such a theory[4]. As in the previous case we will impose a minimal coupling principle. Working in analogy with Brans-Dicke theory, let us define a field $\psi = c^4$, and introduce the following action

$$S = \int dx^4 \left(\sqrt{-g} \left(\frac{\psi(R + 2\Lambda)}{16\pi G} + \mathcal{L}_M \right) + \mathcal{L}_\psi \right) \tag{47}$$

The dynamical variables are a metric $g_{\mu\nu}$, any matter field variables contained in $\mathcal{L}_M$, and ψ itself. The Riemann tensor (and the Ricci scalar) is to be computed from $g_{\mu\nu}$ at constant ψ in the usual way.

As in the previous section covariance is broken, in spite of all appearances. ψ does not appear in coordinate transformations of the metric, and so the connection $\Gamma^\alpha_{\mu\nu}$ does not contain terms in $\nabla\psi$ in any frame. However the connection will contain different terms in ψ in different frames. Hence the statement that the Riemann tensor is to be computed from the metric at constant ψ can only be true in one preferred frame. Minimal coupling requires the definition of a light frame. The action (47) is only Lorentz invariant in appearance.

Varying the action with respect to the metric leads to:

$$\frac{\delta S}{\delta g^{\mu\nu}} = \frac{\sqrt{-g}\psi}{8\pi G}[G_{\mu\nu} - g_{\mu\nu}\Lambda] \tag{48}$$

$$\frac{\delta S_M}{\delta g^{\mu\nu}} = -\frac{\sqrt{-g}\psi}{8\pi G} T_{\mu\nu} \tag{49}$$

leading to a set of Einstein's equations without any extra terms

$$G_{\mu\nu} - g_{\mu\nu}\Lambda = \frac{8\pi G}{\psi} T_{\mu\nu} \tag{50}$$

valid in the light frame. This is the way we chose to phrase our postulates in SectionIII of our paper. In other words all we need is minimal coupling at the level of Einstein's equations.

The fact that a favoured set of coordinates is picked by our action principle is not surprising as Lorentz invariance is broken. On the other hand notice that the dielectric vacuum of Bekenstein theory is an ether theory. His theory also breaks Lorentz invariance, not at the level of the laws of physics, but in the form of the contents of space-time.

In changing α theories a favoured frame is always picked up. In a cosmological setting it makes sense to identify this frame with the cosmological frame. Free falling observers comoving with the cosmological flow define a proper time and a set of spatial coordinates, to be identified with the light frame. In this frame the Einstein-Hilbert action is minimally coupled to a changing ψ, and the same happens to Friedmann equations. The rest of our paper follows.

C. The dynamics of ψ

The definition of $\mathcal{L}_\psi$ controls the dynamics of ψ. This is the most speculative aspect of our theory, but it also opens the doors to empirical model building. In our paper we preferred a scenario in which c changes in an abrupt phase transition, but one could also imagine $c \propto a^n$. The latter scenario would result from a Brans Dicke type of Lagrangian

$$\mathcal{L}_\psi = \frac{-\omega}{16\pi G\psi}\dot{\psi}^2 \tag{51}$$

(where ω is a dimensionless coupling) and is being investigated. Addition of a temperature dependent potential $V(\psi)$ would induce a phase transition, as in the scenario developped in our paper.

However here we only make the following remarks, which are independent of any concrete choice of $\mathcal{L}_\psi$. If $K = \Lambda = 0$ one has

$$\frac{\delta\mathcal{L}_\psi}{\delta\psi} = \frac{\sqrt{-g}T}{4\psi} \tag{52}$$

and so in the radiation dominated epoch ($T = 0$), once $K = \Lambda = 0$, one should not expect driving terms for the ψ equation. Hence once the cosmological problems are solved, in the radiation epoch, c and $\hbar$ should be constants. Incidently, once the matter dominated epoch ($T \neq 0$) is reached, ψ should perhaps start changing again, with interesting observations consequences [22]. We are studying the phase space portraits of these cosmologies, when say $\Lambda \neq 0$, and with various $\mathcal{L}_\psi$.

During phase transitions the perfect fluid approximation must break down. One should then use, say, scalar field theory (let's call it ϕ). Now notice that terms in $\dot{\phi}$ will act as a source to ψ (as they contain the speed of light). Hence whenever there is a phase transition and the VEV of a field changes a large amount, one may expect a large change in the speed of light, with most choices of $\mathcal{L}_\psi$. A changing ψ associated with SSB could then solve the quantum version of the cosmological constant problem, but this might require a rather contorted choice of $\mathcal{L}_\psi$.

D. Axiomatic formulation of VSL theories

Postulate 1. *A changing α is to be interpreted as a changing c and $\hbar$ in the ratios $c \propto \hbar \propto \alpha^{-1/2}$. The coupling e is constant.*

[4] Gravitation is normally regarded as the gauge theory of the Poincare group [24]. Here we simply abandon this point of view. In some future work we will try to define a gauge principle for broken symmetries, thereby recovering the standard view

This postulate merely sets up the theoretical interpretation of the possible experimental fact that α changes, in terms of variable dimensional quantities. This is a matter of convention and not experiment, as much as a constant $\hbar c$ is a matter of convention. With the above choice a system of units for mass, length, time, and temperature is unambiguously defined.

Postulate 2. *There is a preferred frame for the laws of physics. This preferred frame is normally suggested by the symmetries of the problem, or by a criterium such as* $c = c(t)$.

If c is variable, Lorentz invariance must be broken. Even if one writes Lorentz invariant looking expressions these do not transform covariantly. [In general this boils down to the explicit presence of c in the operator ∂_μ. Once one admits that Lorentz invariance must be explicitly broken then a preferred frame must exist to formulate the laws of physics. These laws are not invariant under frame transformation, and one may expect that a preferred frame exists where these laws simplify.

Postulate 3. *In the preferred frame one may obtain the laws of physics simply by replacing c in the standard (Lorentz invariant) action, wherever it occurs, by a field* $c = c(x^\mu)$.

This is the principle of minimal coupling. Because the laws of physics cannot be Lorentz invariant it will not hold in every frame. Hence the application of this postulate depends crucially on the previous postulate supplying us with a favoured frame. This principle may apply in Minkowski space time electrodynamics, scalar field theory, etc, in which case the frame in which $c = c(t)$ is probably the best choice. The cosmological frame, endowed with the cosmic proper time is probably the best choice in a cosmological setting.

Postulate 4 *The dynamics of c must be determined by an action principle deriving from adding an extra term to the Lagrangian which is a function of c only.*

This is work in progress. We do not wish to specify this postulate further because for all we know this extra term can be anything. We merely specify that no fields (including the metric) must be present in this extra term because we wish minimal coupling to propagate into the Einstein's equations.

APPENDIX II: SCALAR PERTURBATION EQUATIONS FOR VSL MODELS

In this Appendix we derive the scalar cosmological perturbation equations in VSL scenarios. We assume $K = \Lambda = 0$, and use a gauge where the perturbed metric is written as

$$ds^2 = a^2[-(1+2AY)d\eta^2 - 2BYk_i dx^i d\eta + \delta_{ij}dx^i dx^j] \tag{53}$$

for a Fourier component with wave vector k^i. Here Y is a scalar harmonic. We shall use conformal time η to study fluctuations, and denote $' = d/d\eta$. The stress energy tensor is also writen as

$$\begin{aligned} \delta T^0_0 &= -\rho Y \delta \\ \delta T^i_0 &= -\Big(\rho + \frac{p}{c^2}\Big)\frac{v}{c}k^i Y \\ \delta T^i_j &= p\Pi_L Y \delta^i_j + (k^i k_j - 1/3\delta^i_j k^2) Y p \Pi_T \end{aligned} \tag{54}$$

The Einstein's constraint equations then read [13]

$$\frac{3}{c^2}\left(\frac{a'}{a}\right)^2 A - \frac{1}{c}\frac{a'}{a}kB = -\frac{4\pi G a^2}{c^2}\rho\delta \tag{55}$$

$$\frac{k}{c}\frac{a'}{a}A - \left(\left(\frac{a'}{a}\right)' - \left(\frac{a'}{a}\right)^2\right)\frac{B}{c^2} = \frac{4\pi G a^2}{c^2}\Big(\rho + \frac{p}{c^2}\Big)\frac{v}{c} \tag{56}$$

and the dynamical equations are

$$A + \frac{1}{kc}\left(B' + 2\frac{a'}{a}B\right) = -\frac{8\pi G a^2}{c^2}\frac{p}{c^2}\frac{\Pi_T}{k^2} \tag{57}$$

$$\frac{a'}{a}\frac{A'}{c^2} + \left(2\frac{a''}{a} - \left(\frac{a'}{a}\right)^2\right)\frac{A}{c^2} = \frac{4\pi G a^2}{c^2}\frac{p}{c^2}(\Pi_L - 2\Pi_T) \tag{58}$$

We assume that these equations do not receive corrections in $\dot{c}/c$. This statement is gauge-dependent, much like its counterpart for the unperturbed Eintein's equations. We can only hope that the physical result does not change qualitatively from gauge to gauge. Complying with tradition we now define the comoving density contrast [13]

$$\Delta = \delta + 3(1+w)\frac{a'}{ca}\frac{1}{k}\Big(\frac{v}{c} - B\Big) \tag{59}$$

We also introduce the entropy production rate

$$\Gamma = \Pi_L - \frac{c_s^2}{wc^2}\delta \tag{60}$$

where the speed of sound c_s is given by

$$c_s^2 = \frac{p'}{\rho'} = wc^2\left(1 - \frac{2}{3}\frac{1}{1+w}\frac{c'}{c}\frac{a}{a'}\right) \tag{61}$$

Note that the thermodynamical speed of sound is given by $c_s^2 = (\partial p/\partial\rho)|_S$. Since in SBB models evolution is isentropic $c_s^2 = (\partial p/\partial \rho)|_S = \dot{p}/\dot{\rho} = p'/\rho'$. When $\dot{c} \neq 0$ evolution need not be isentropic. However we keep the definition $c_s^2 = p'/\rho'$ since this is the definition used in perturbative calculations. One must however remember that the speed of sound given in (61) is not the usual thermodynamical quantity. With this definition one has for adiabatic perturbations $\delta p/\delta\rho = p'/\rho'$, that is the ratio between pressure and density fluctuations mimics the ratio of their background rates of change.

Combining all four Einstein's equations we can then obtain the (non-)conservation equations [13]

$$\Delta' - \left(3w\frac{a'}{a} + \frac{c'}{c}\right)\Delta = -(1+w)kv - 2\frac{a'}{a}w\Pi_T \tag{62}$$

$$v' + \left(\frac{a'}{a} - 2\frac{c'}{c}\right)v = \left(\frac{c_s^2 k}{1+w} - \frac{3}{2k}\frac{a'}{a}\left(\frac{a'}{a} + \frac{c'}{c}\right)\right)\Delta + \frac{kc^2 w}{1+w}\Gamma - kc\left(\frac{2/3}{1+w} + \frac{3}{k^2c^2}\left(\frac{a'}{a}\right)^2\right)w\Pi_T \tag{63}$$

These can then be combined into a second order equation for Δ. If $\Gamma = \Pi_T = 0$ this equation takes the form

$$\Delta'' + f\Delta' + (g + h + c_s^2k^2)\Delta = 0 \tag{64}$$

with

$$f = (1-3w) - 3\frac{c'}{c} \tag{65}$$

$$g = \left(\frac{a'}{a}\right)^2\left(\frac{9}{2}w^2 - 3w - \frac{3}{2}\right) \tag{66}$$

$$h = 2\left(\frac{c'}{c}\right)^2 + \left(\frac{9w}{2} - \frac{5}{2}\right)\frac{c'}{c}\frac{a'}{a} - \left(\frac{c'}{c}\right)' \tag{67}$$

APPENDIX III: VECTOR PERTURBATION EQUATIONS FOR VSL MODELS

In a similar fashion we can study vector modes in a gauge where the metric may be written as

$$ds^2 = a^2[-d\eta^2 + 2BY_i dx^i d\eta + \delta_{ij}dx^i dx^j] \tag{68}$$

where Y_i is a vector harmonic. The stress energy tensor is written as

$$\delta T_i^0 = \left(\rho + \frac{p}{c^2}\right)\left(\frac{v}{c} - B\right)Y_i \tag{69}$$

$$\delta T_{ij} = p\Pi^T Y_{(i,j)} \tag{70}$$

Einstein's equations then read [13]

$$k^2 B = \frac{16\pi G}{c^2}a^2\left(\rho + \frac{p}{c^2}\right)\frac{v}{c} \tag{71}$$

$$\frac{k}{c}\left(B' + 2\frac{a'}{a}B\right) = -\frac{8\pi G}{c^2}\frac{p}{c^2}a^2\Pi^T \tag{72}$$

We assume that these do not receive $\dot{c}/c$ corrections. The conservation equation is then:

$$v' + (1-3w)\frac{a'}{a}v - 2\frac{c'}{c}v = -\frac{kc}{2}\frac{w}{1+w}\Pi^T \tag{73}$$

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